Sec 2.6 Geometry – Triangle Proofs

Name:

COMMON POTENTIAL REASONS FOR PROOFS

Definition of Congruence: Having the exact same size and shape and there by having the exact same measures.
Definition of Midpoint: The point that divides a segment into two congruent segments.
Definition of Angle Bisector: The ray that divides an angle into two congruent angles.
Definition of Perpendicular Lines: Lines that intersect to form right angles or 90°
Definition of Supplementary Angles: Any two angles that have a sum of 180°
Definition of a Straight Line: An undefined term in geometry, a line is a straight path that has no thickness and extends forever. It also forms a straight angle which measures 180°

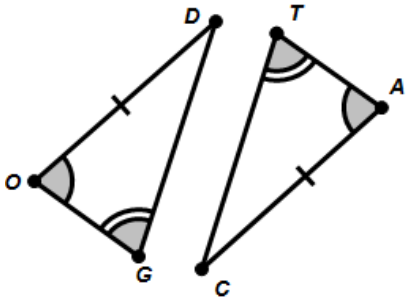
Reflexive Property of Equality: any measure is equal to itself (a = a)
Reflexive Property of Congruence: any figure is congruent to itself ($Figure\ A \cong Figure\ A$)
Addition Property of Equality: if a = b, then a + c = b + c
Subtraction Property of Equality: if a = b, then a – c = b – c
Multiplication Property of Equality: if a = b, then ac = bc
Division Property of Equality: if a = b, then $\frac{a}{c} = \frac{b}{c}$
Transitive Property: if a = b & b =c then a = c OR if a ≅ b & b ≅ c then a ≅ c.

Segment Addition Postulate: If point B is between Point A and C then AB + BC = AC
Angle Addition Postulate: If point S is in the interior of ∠PQR, then $m\angle PQS + m\angle SQR = m\angle PQR$
Side – Side – Side Postulate (SSS) : If three sides of one triangle are congruent to three sides of another triangle, then the triangles are congruent.
Side – Angle – Side Postulate (SAS): If two sides and the included angle of one triangle are congruent to two sides and the included angle of another triangle, then the triangles are congruent.
Angle – Side – Angle Postulate (ASA): If two angles and the included side of one triangle are congruent to two angles and the included side of another triangle, then the triangles are congruent.
Angle – Angle – Side Postulate (AAS) : If two angles and the non-included side of one triangle are congruent to two angles and the non-included side of another triangle, then the triangles are congruent
Hypotenuse – Leg Postulate (HL): If a hypotenuse and a leg of one right triangle are congruent to a hypotenuse and a leg of another right triangle, then the triangles are congruent

Right Angle Theorem (R.A.T.): All right angles are congruent.
Vertical Angle Theorem (V.A.T.): Vertical angles are congruent.
Triangle Sum Theorem: The three angles of a triangle sum to 180°
Linear Pair Theorem: If two angles form a linear pair then they are adjacent and are supplementary.
Third Angle Theorem: If two angles of one triangle are congruent to two angles of another triangle, then the third pair of angles are congruent.
Alternate Interior Angle Theorem (and converse): Alternate interior angles are congruent if and only if the transversal that passes through two lines that are parallel.
Alternate Exterior Angle Theorem (and converse): Alternate exterior angles are congruent if and only if the transversal that passes through two lines that are parallel.
Corresponding Angle Theorem (and converse) : Corresponding angles are congruent if and only if the transversal that passes through two lines that are parallel.
Same-Side Interior Angles Theorem (and converse) : Same Side Interior Angles are supplementary if and only if the transversal that passes through two lines that are parallel.
Pythagorean Theorem (and converse): A triangle is right triangle if and only if the given the length of the legs a and b and hypotenuse c have the relationship $a^2+b^2 = c^2$
Isosceles Triangle Theorem (and converse): A triangle is isosceles if and only if its base angles are congruent.
Triangle Mid-segment Theorem: A mid-segment of a triangle is parallel to a side of the triangle, and its length is half the length of that side.

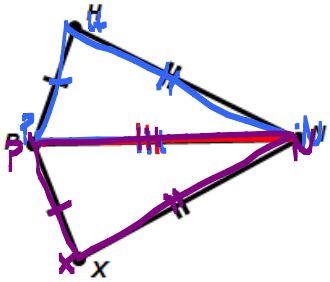
CPCTC**:** Corresponding Parts of Congruent Triangles are Congruent by definition of congruence.

1. Tell which of the following triangle provide enough information to show that they must be congruent. If they are congruent, state which theorem suggests they are congruent (SAS, ASA, SSS, AAS, HL) and write a congruence statement.



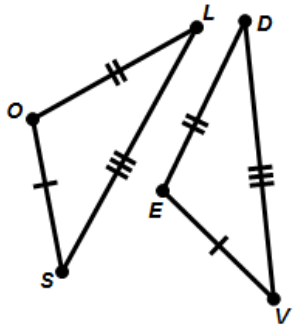
Circle one of the following:
SSS SAS ASA **AAS** HL Not Enough Information

Congruence Statement if necessary: $\Delta DOG \cong \Delta CAT$



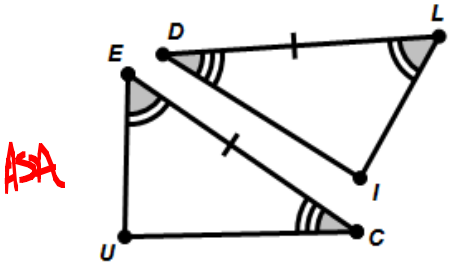
Circle one of the following:
SSS SAS ASA AAS HL Not Enough Information

Congruence Statement if necessary: $\Delta PHN \cong \Delta PXN$



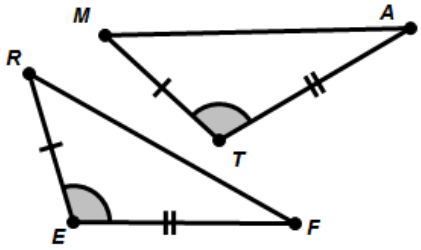
Circle one of the following:
SSS SAS ASA AAS HL Not Enough Information

Congruence Statement if necessary: $\Delta SOL \cong \Delta VED$



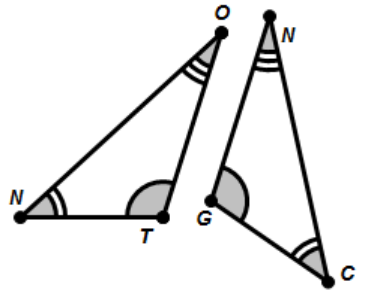
Circle one of the following:
SSS SAS **ASA** AAS HL Not Enough Information

Congruence Statement if necessary: $\Delta EVC \cong \Delta LID$



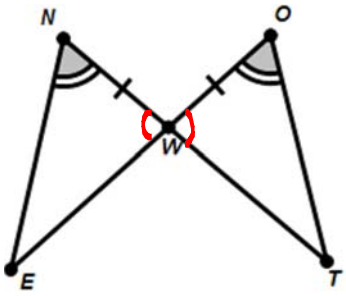
Circle one of the following:
SSS **SAS** ASA AAS HL Not Enough Information

Congruence Statement if necessary: $\Delta FER \cong \Delta ATM$



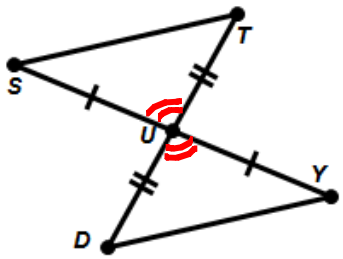
Circle one of the following:
SSS SAS ASA AAS HL **Not Enough Information**

Congruence Statement if necessary:



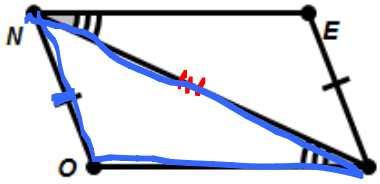
Circle one of the following:
SSS SAS **ASA** AAS HL Not Enough Information

Congruence Statement if necessary: $\Delta NEW \cong \Delta OTW$



Circle one of the following:
SSS **SAS** ASA AAS HL Not Enough Information

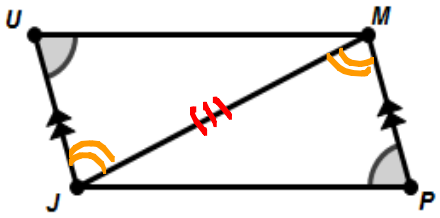
Congruence Statement if necessary: $\Delta STU \cong \Delta YDU$



CAN'T USE ~~AAA~~ AND ~~SSA~~

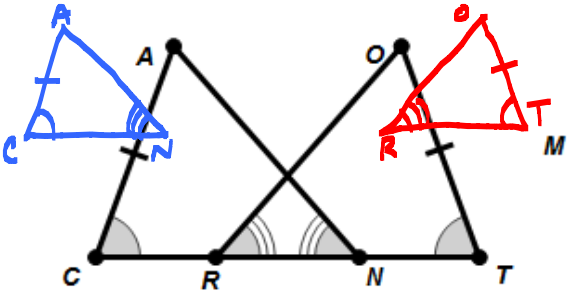
Circle one of the following:
SSS SAS ASA AAS HL **Not Enough Information**

Congruence Statement if necessary: SSA



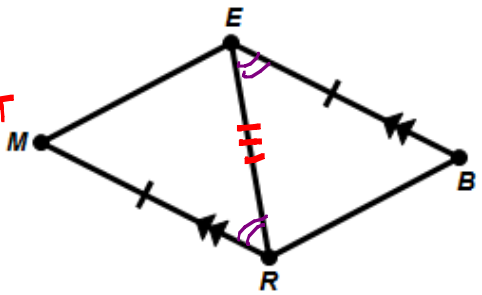
Circle one of the following:
SSS SAS ASA **AAS** HL Not Enough Information

Congruence Statement if necessary: $\Delta JUM \cong \Delta MPJ$



Circle one of the following:
SSS SAS ASA **AAS** HL Not Enough Information

Congruence Statement if necessary: $\Delta CAN \cong \Delta TOR$



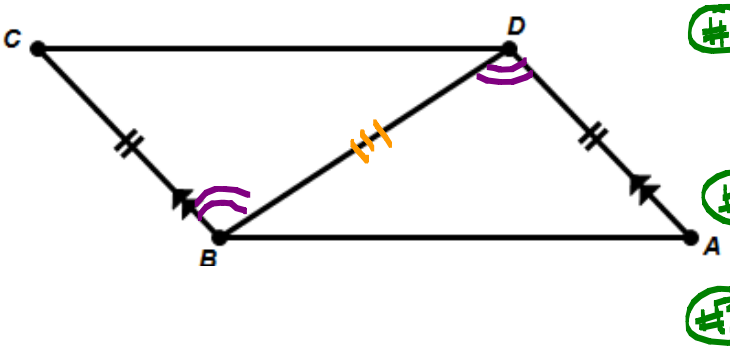
Circle one of the following:
SSS **SAS** ASA AAS HL Not Enough Information

Congruence Statement if necessary: $\Delta REM \cong \Delta ERB$

$\Delta MER \cong \Delta BRE$

2. Prove which of the following triangles congruent if possible by filling in the missing blanks:

a. Given $\overline{CB} \cong \overline{AD}$ and $\overleftrightarrow{CB} \parallel \overleftrightarrow{AD}$



Statements	Reasons
1. $\overline{CB} \cong \overline{AD}$	GIVEN
2. $\overleftrightarrow{CB} \parallel \overleftrightarrow{AD}$	GIVEN
3. $\measuredangle CBD \cong \measuredangle ADB$	A.I.A. ARE $\cong$
4. $\overline{BD} \cong \overline{BD}$	REFLEXIVE PROP. OF CONGRUENCE
5. $\Delta BCD \cong \Delta DAB$	S.A.S., 1,3,4

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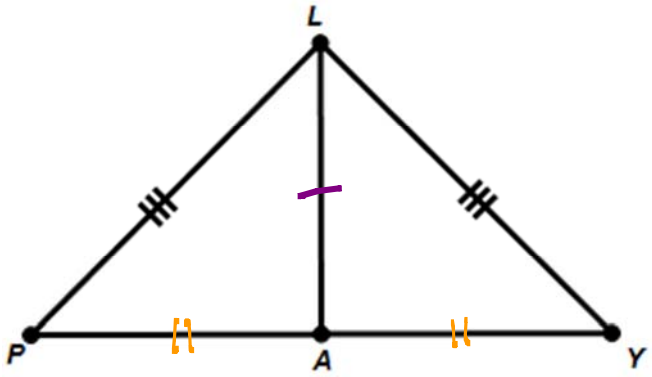
b. Given $\overline{PL} \cong \overline{YL}$ and Point A is the midpoint of $\overline{PY}$



Statements	Reasons
1. $\overline{PL} \cong \overline{YL}$	Given
2. POINT A IS MIDPOINT OF $\overline{PY}$	Given
3. $\overline{PA} \cong \overline{YA}$	Definition of Midpoint
4. $\overline{LA} \cong \overline{LA}$	Reflexive property of congruence
5. $\Delta PAL \cong \Delta YAL$	By steps 1,3,4 and SSS

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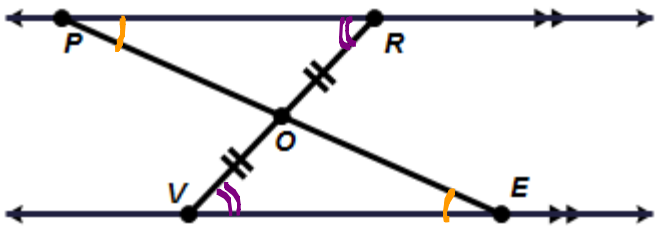
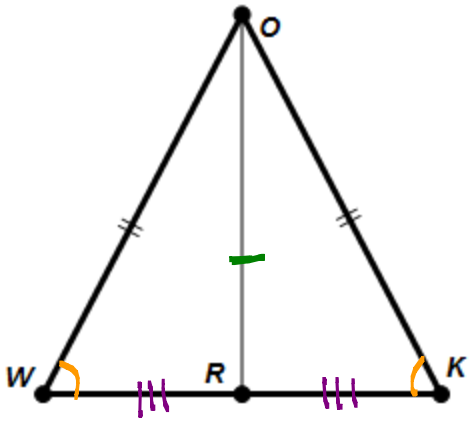
c. Given $\overline{VO} \cong \overline{RO}$ and $\overleftrightarrow{PR} \parallel \overleftrightarrow{VE}$



Statements	Reasons
1. $\overline{VO} \cong \overline{RO}$	Given
2. $\overleftrightarrow{PR} \parallel \overleftrightarrow{VE}$	Given
3. $\measuredangle PRO \cong \measuredangle EVO$	AIA ARE $\cong$
4. $\measuredangle RPO \cong \measuredangle VEO$	AIA ARE $\cong$
5. $\Delta PRO \cong \Delta EVO$	AAS (STEP 1, 3,4)

Prove the Isosceles Triangle Theorem and the rest of the suggested proofs.

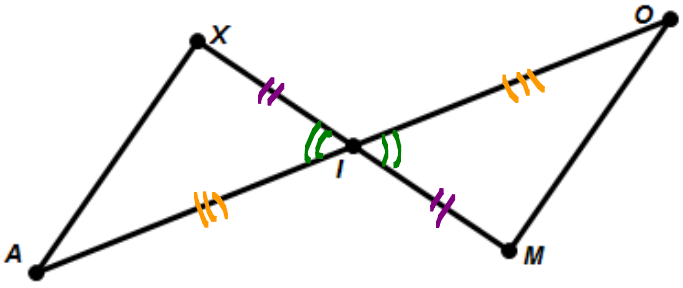
d. Given ΔWOK is isosceles and point R is the midpoint of $\overline{WK}$



Statements	Reasons
1. ΔWOK is isosceles	GIVEN
2. $\overline{WO} \cong \overline{KO}$	DEFINITION OF AN ISOSCELES TRIANGLE
3. R is the midpoint of $\overline{WK}$	GIVEN
4. $\overline{WR} \cong \overline{KR}$	DEFINITION OF A MIDPOINT
5. $\overline{OR} \cong \overline{OR}$	REFLEXIVE PROP. OF CONGRUENCE
6. $\Delta WRO \cong \Delta KRO$	SSS (STEP 2, 4, 5)
7. $\measuredangle OWR \cong \measuredangle OKR$	CPCTC

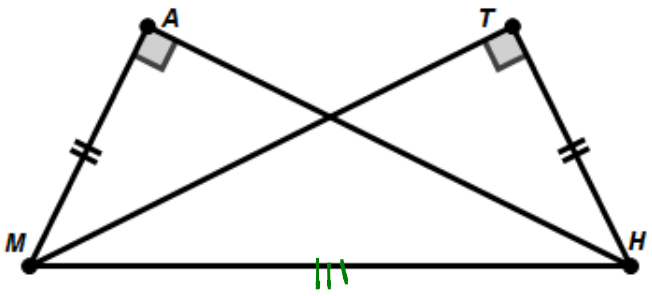
CPCTC: CORRESPONDING PARTS OF CONGRUENT TRIANGLES ARE CONGRUENT.

e. Given point I is the midpoint of $\overline{XM}$ and point I is the midpoint of $\overline{AO}$



Statements	Reasons
1. I is the midpoint of $\overline{XM}$	GIVEN
2. $\overline{XI} = \overline{MI}$	Definition of Midpoint
3. I is the midpoint of $\overline{AO}$	GIVEN
4. $\overline{AI} \cong \overline{OI}$	DEF. OF MIDPOINT
5. $\measuredangle AIX \cong \measuredangle OIM$	VERTICAL ANGLE THEOREM
6. $\Delta AXI \cong \Delta OMI$	S.A.S. (STEP. 2, 4, 5)

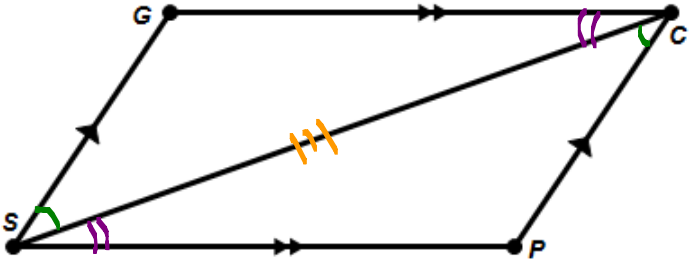
f. Given $\measuredangle MAH$ and $\measuredangle HTM$ are right angles and $\overline{MA} \cong \overline{TH}$



Statements	Reasons
1. $\measuredangle MAH$ & $\measuredangle HTM$ are right angles	GIVEN
2. $\overline{MA} \cong \overline{TH}$	GIVEN
3. $\overline{MH} \cong \overline{MH}$	REFLEXIVE PROP. OF CONGRUENCE
4. $\Delta MAH \cong \Delta HTM$	HL (STEP 2, 3)

Prove the suggested proofs by filling in the missing blanks.

g. Given $\overleftrightarrow{GC} \parallel \overleftrightarrow{PS}$ and $\overleftrightarrow{GS} \parallel \overleftrightarrow{CP}$



Statements	Reasons
1. $\overleftrightarrow{GC} \parallel \overleftrightarrow{PS}$	GIVEN
2. $\angle GCS \cong \angle PSC$	AIA ARE $\cong$
3. $\overleftrightarrow{GS} \parallel \overleftrightarrow{CP}$	GIVEN
4. $\angle GSC \cong \angle PCS$	AIA ARE $\cong$
5. $\overline{SC} \cong \overline{SC}$	REFLEXIVE PROP.
6. $\Delta GCS \cong \Delta PSC$	ASA (STEP 2, 4, 5)

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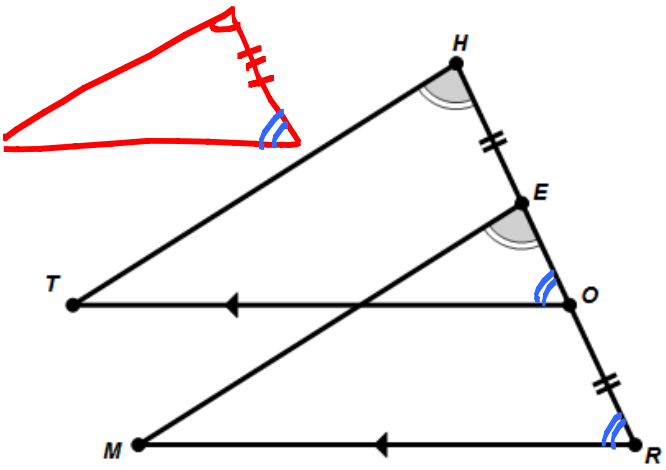
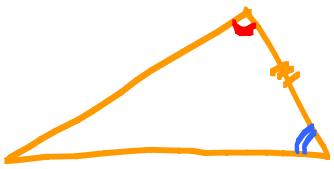
h. Given $\overline{RO} \cong \overline{HE}$, $\measuredangle THO \cong \measuredangle MER$ and $\overleftrightarrow{TO} \parallel \overleftrightarrow{MR}$

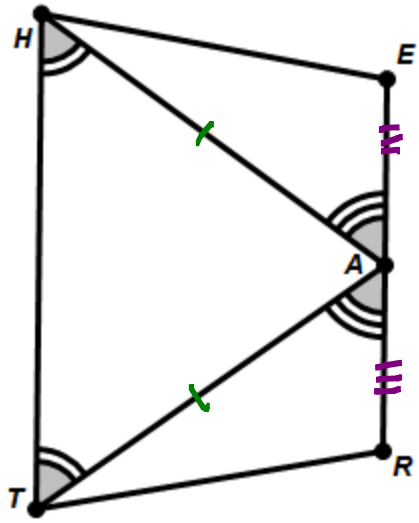




Statements	Reasons
1. $\overline{HE} \cong \overline{OR}$	GIVEN
2. $RO = HE$	DEFINITION OF CONGRUENCE
3. $RO + OE = HE + EO$	ADDITION PROPERTY OF EQUALITY
4. $RO + OE = RE$ and $HE + EO = HO$	SEGMENT ADDITION POSTULATE
5. $RE = HO$	Substitution Property
6. $\overline{RE} \cong \overline{OR}$	DEFINITION OF CONGRUENCE
7. $\overleftrightarrow{TO} \parallel \overleftrightarrow{MR}$	GIVEN
8. $\measuredangle MRE \cong \measuredangle TOH$	CORRESPONDING ANGLES $\cong$
9. $\measuredangle THO \cong \measuredangle MER$	GIVEN
10. $\Delta THO \cong \Delta MER$	ASA (STEPS 5, 8, 9)

Prove the suggested proofs by filling in the missing blanks.

i. Given ∡HTA ≅ ∡THA , ∡TAR ≅ ∡HAE, and Point A is the midpoint of $\overline{ER}$



Statements	Reasons
1. $\measuredangle THA \cong \measuredangle HTA$	GIVEN
2. ΔHAT is an isosceles triangle	CONVERSE OF THE ISOSCELES Δ THM & DEF.
3. $\overline{HA} \cong \overline{TA}$	DEFINITION OF ISOSCELES TRIANGLE
4. A is the midpoint of $\overline{ER}$	GIVEN
5. $\overline{EA} \cong \overline{AR}$	DEFINITION OF MIDPOINT
6. $\measuredangle TAR \cong \measuredangle HAE$	GIVEN
7. $\Delta TAR \cong \Delta HAE$	SAS (STEP 3, 5, 6)

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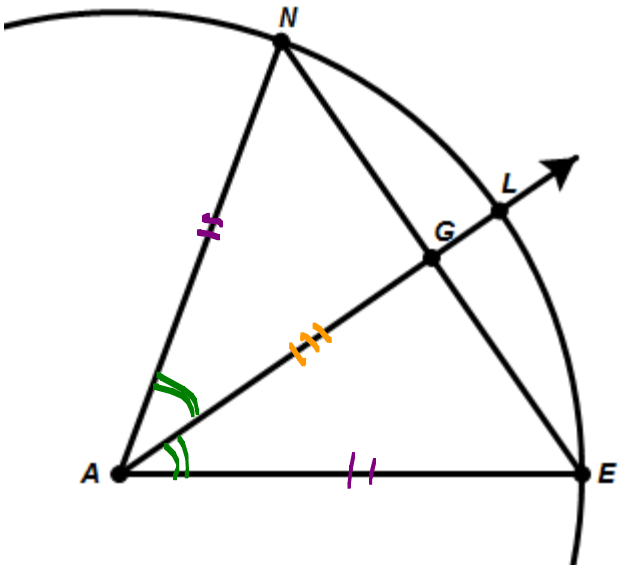
j. Given that $\overrightarrow{AG}$ bisects ∡NAE. Also, $\overline{AN}$ and $\overline{AE}$ are radii of the same circle with center A.



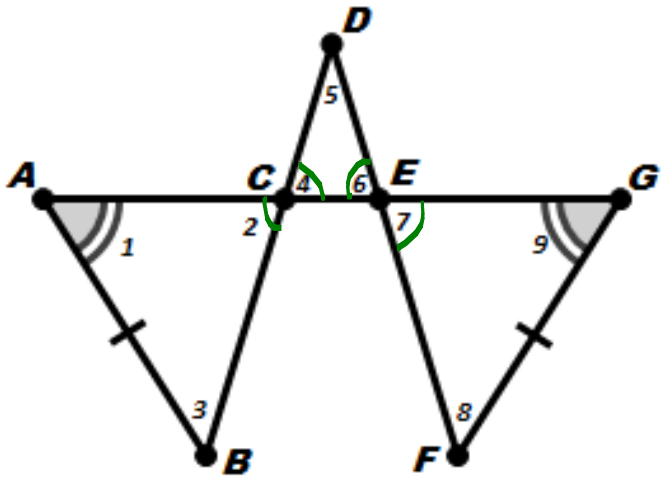
Statements	Reasons
1. $\overrightarrow{AG}$ bisects $\measuredangle NAE$	GIVEN
2. ∡NAG ≅ ∡EAG	Definition of Angle Bisector
3. $\overline{AN} \cong \overline{AE}$	Radii of the same circle are congruent.
4. $\overline{AG} \cong \overline{AG}$	Reflexive property of congruence
5. $\Delta ANG \cong \Delta AEG$	SAS (STEP 2, 3, 4)

Prove the suggested proofs by filling in the missing blanks.

i. **Given:**

- $\measuredangle 1 \cong \measuredangle 9$
- $\overline{AB} \cong \overline{GF}$
- ΔCDE forms an isosceles triangle with base $\overline{CE}$

Prove: $\Delta ABC \cong \Delta GFE$



Statements	Reasons
1. $\measuredangle 1 \cong \measuredangle 9$	GIVEN
2. $\overline{AB} \cong \overline{GF}$	GIVEN
3. $\measuredangle 2 \cong \measuredangle 4$	V. A. T.
4. ΔCDE is an isosceles triangle	GIVEN
5. $\measuredangle 4 \cong \measuredangle 6$	Isosceles Triangle Theorem
6. $\measuredangle 6 \cong \measuredangle 7$	V. A. T
7. $\measuredangle 2 \cong \measuredangle 7$	TRANSITIVE PROP
8. $\Delta ABC \cong \Delta GFE$	AAS. STEP (1,2,7)

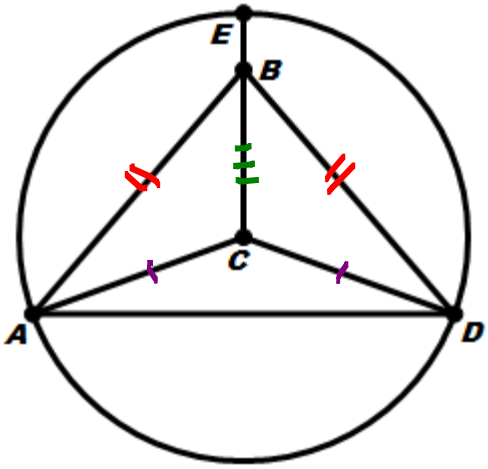
$\measuredangle 2 \cong \measuredangle 4 \cong \measuredangle 4 \cong \measuredangle 6 \cong \measuredangle 6 \cong \measuredangle 7$

Prove the suggested proofs by filling in the missing blanks.

k. **Given:**

- **The circle has a center at point C**
- ΔABD forms an isosceles triangle with base $\overline{AD}$

Prove: $\Delta ABC \cong \Delta DBC$



Statements	Reasons
1. ΔABD is an isosceles triangle w/ base $\overline{AD}$	GIVEN
2. $\overline{AE} \cong \overline{DB}$	DEF. OF ISOSCELES TRIANGLE
3. The circle is centered at point C	GIVEN
4. $\overline{AC} \cong \overline{DC}$	RADII OF SAME CIRCLE ARE $\cong$
5. $\overline{BC} \cong \overline{BC}$	Reflexive Property of Congruence
6. $\Delta ABC \cong \Delta DBC$	SSS (STEP 2,4,5)