Measuring wave celerity in the West Branch of the Susquehanna River using cross-correlation of stage data for adjacent stream gauges

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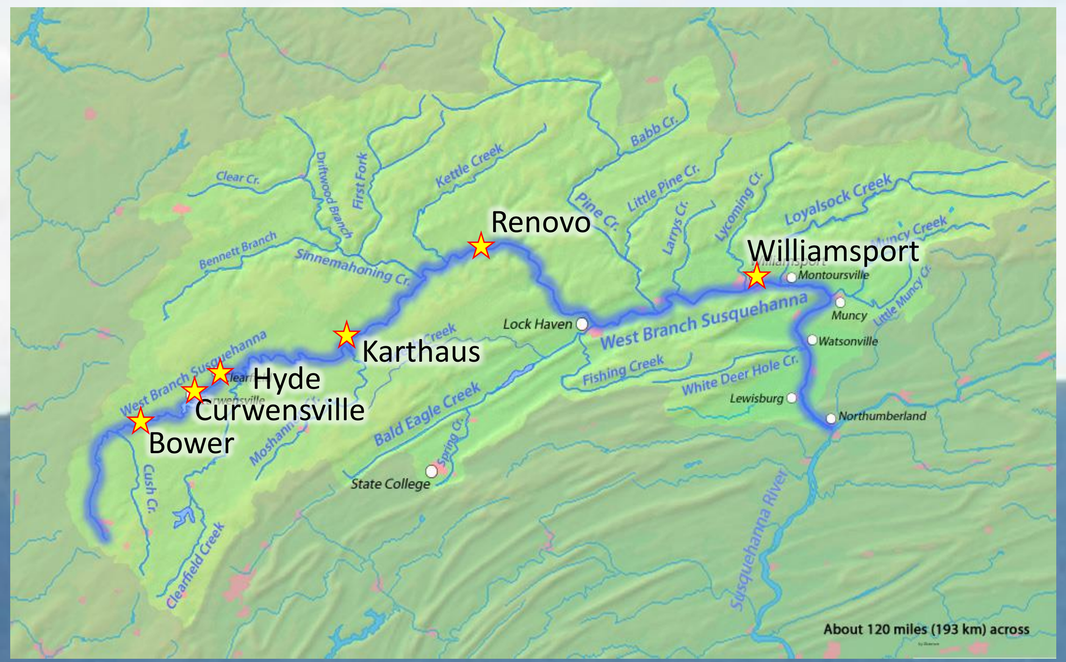
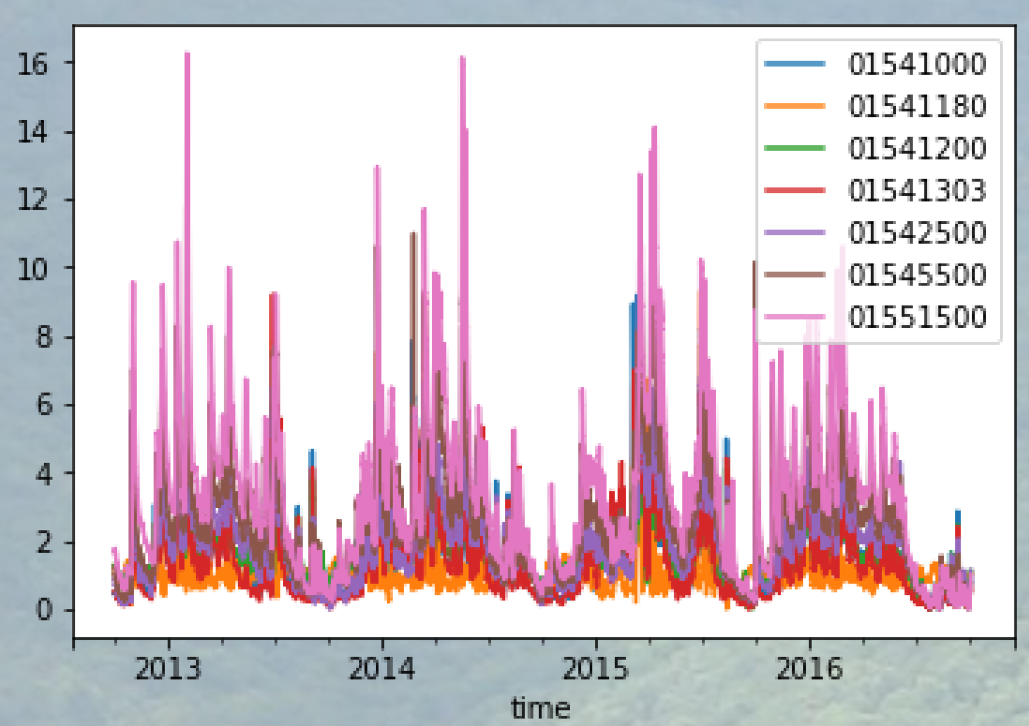


	USGS ID#	Drainage km^2	Median Q cfs	Gauge Datum	Zero flow stage
Bower	01541000	816	114	1207.14	3.88
Curwensville	01541200	951	149	1124.66	2.51
Hyde	01541303	1,228	214	1093.90	2.55
Karthaus	01542500	3,787	610	830.59	0.64
Renovo	01545500	7,705	7400	634.19	- 0.68
Williamsport	01551500	14,716	14600	494.98	- 0.88

Table 1. The stream gauges used in this study, plus four nearby stream gauges (in parentheses) that were not used in this study.

Study Site & Data:

Four years of 15-minute stage & discharge data from a series of six USGS stream gauges along the West Branch of the Susquehanna were downloaded and processed using HydroFunctions, a Python package for analyzing hydrology data.



Abstract

Celerity expresses the speed of a wave crossing a water surface. In rivers, waves typically travel faster than the velocity of the water. In this study, I adapted cross-correlation techniques from coastal wave research and applied them to flood waves in the West Branch of the Susquehanna River using the HydroFunctions Python package. Four years of 15-minute river stage and discharge data captured flood waves as they passed a series of US Geological Survey stream gages. Shorter, eleven-day slices of this data were lagged successively; the lag with the highest correlation to the downstream record was considered a match and used to calculate wave celerity. Unfortunately, flood waves are more irregular in shape, amplitude, and period than coastal waves and require pre-processing to improve the matching efficiency. A second-order, forward-backward, one-day high-pass Butterworth filter produced better results than the use of either raw values or differenced values in the cross-correlation analysis. The use of stage data produced matches as efficiently as the use of discharge data. Ten randomly-selected waves were also tracked manually to serve as a validation data set.

Results indicate the expected positive, linear relationship between stage and celerity when considering a single reach, but surprisingly, this relationship breaks down when individual waves are tracked downstream, longitudinally. Celerity and discharge typically increase in downstream reaches, while stage may or may not. The ability to quickly produce empirical measurements of wave celerity allows the calibration of flood routing models and aid in model selection.

What is Celerity?

Celerity is the speed of a wave crest as it travels through the water. Velocity is the speed of the water itself. If you were floating in a reservoir and a boat passed by, the speed of the wake would be the wave's celerity. The water velocity would be zero.

In most natural streams, wave celerity is greater than the water velocity. However, in whitewater or downstream from a low-head dam, can get situations where the water velocity is faster than the wave celerity. As the water rushes forward, it gradually slows down and eventually reaches a critical point where the two speeds are identical. This produces a standing wave.

Methods:

1. **Validation dataset**
   - a. Randomly select 10 waves
   - 1b. Track waves as they move downstream.
   - c. Measure how long peaks take to travel downstream
   - d. Calculate celerity *(celerity = distance / travel time)*
2. **Analysis dataset**
   - 2a. Transform stage data by removing 'slow' changes (run 1-day high-pass filter)
   - b. Split data into shorter, 11-day chunks
   - 2c. Use cross-correlation to measure travel time
   - d. Calculate celerity

Results:

3. Cross-correlation can produce results similar to manually tracking peaks.

Unfortunately, approximately 10% of the segments appear to get mis-matched.

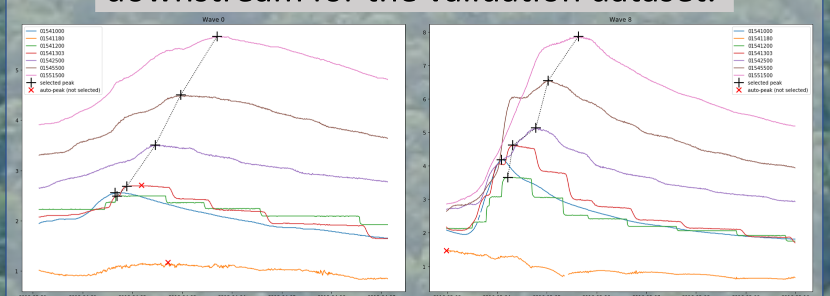
4. Stage & Celerity "at-a-Station": Deeper waves move faster.

There is a linear, direct relationship between stage (y in the Saint-Venant equations) and celerity. The relationship between discharge (Q) and celerity is non-linear.

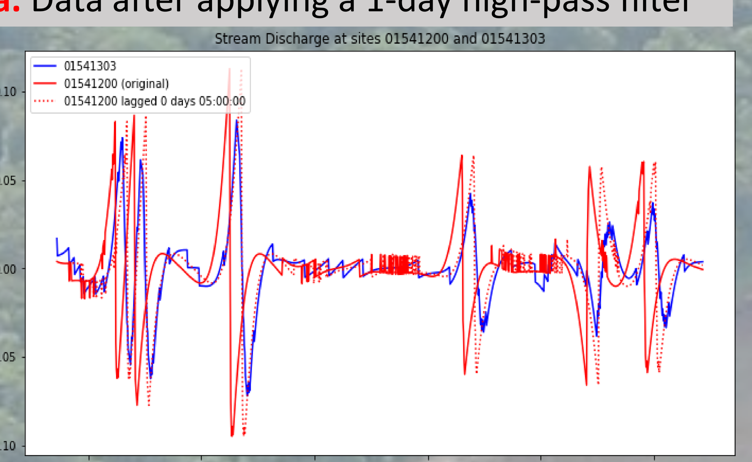
5. Downstream: Celerity increases; stage & discharge unrelated

Most waves increase in celerity and discharge as they travel downstream, but their stage stays the same. Another group of waves increase in stage as they travel downstream; these waves do not fit the stage-celerity relationship as well. These waves increase in size due to water from tributaries (q in the Saint-Venant equations). The smallest group of waves decreases in discharge, stage, and celerity as they travel downstream.

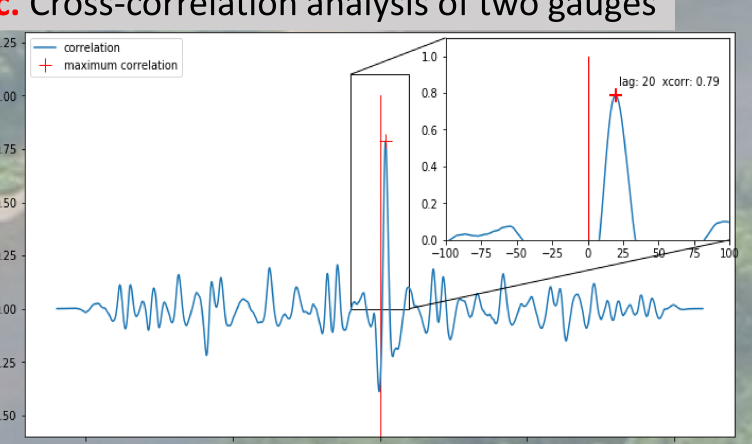
1b. Tracking two waves as they travel downstream for the validation dataset.



2a. Data after applying a 1-day high-pass filter

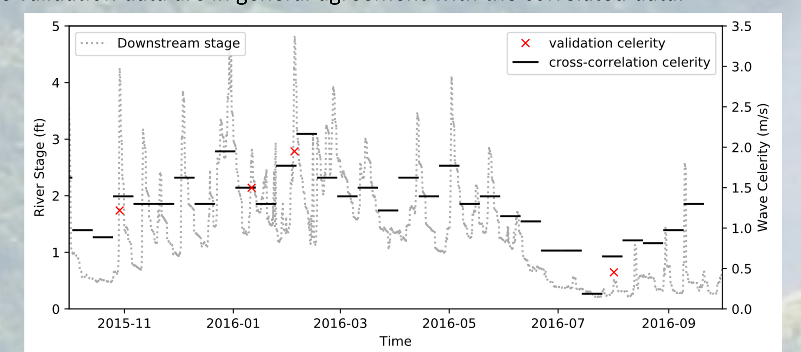


2c. Cross-correlation analysis of two gauges



3. Does cross-correlation produce accurate estimates of celerity?

A one year plot of stream stage at the Curwensville and Hyde stations (left axis), with celerity for the reach (right axis) over-printed. The celerity validation data are plotted with 'X' symbols at the time of the wave peak, while the cross-correlation values are plotted as horizontal bars for the length of the correlated time series. Note how a higher stage tends to correspond with a higher celerity, and the validation data are in general agreement with the correlated data.



4. Stage & Celerity "at-a-station":

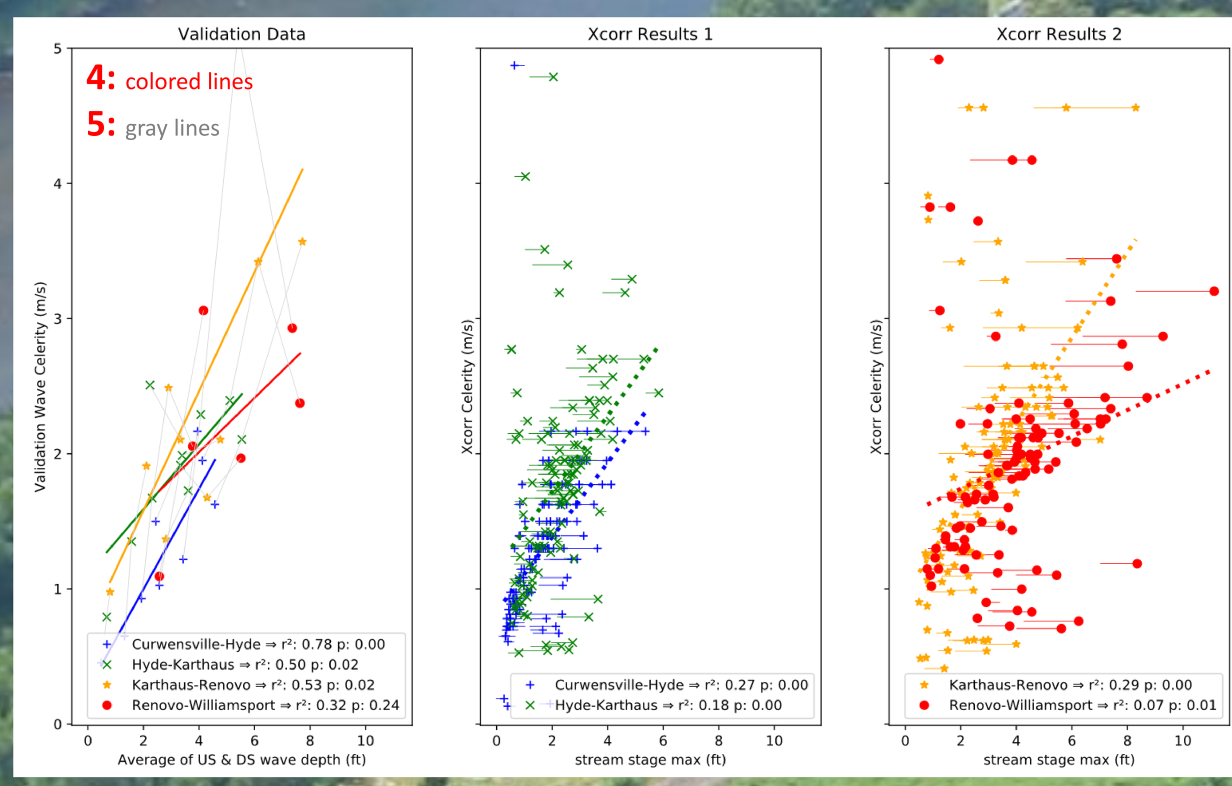
There is a fairly linear, direct relationship at each station between stage and celerity.

Validation dataset: Variations in the average of the upstream and downstream wave depth (X-axis) explains 32-78% of the variation in wave celerity (Y-axis) when considered at a station (colored lines).

Cross-Correlation dataset: Celerity (y-axis) plotted against stage (x-axis) for two adjacent reaches. Each observation is plotted as a line connecting the maximum stage at the upstream station to the downstream station, with the downstream station marked with a point symbol. Poor matches during the cross-correlation procedure result in much lower r^2 values, but the slopes and intercepts for each reach are similar to the values for the validation dataset.

5. Stage & Celerity downstream:

Unfortunately, it is not as easy to predict the changes in wave depth and celerity for individual waves as they move downstream (gray lines, left diagram). Most waves that are tracked downstream tend to stay the same stage while their celerity increases. This is also reflected in the reach regression lines: downstream reaches have higher celerities for the same range of stages.



Conclusions:

- ❖ Water from 'side' tributaries plays an important role in determining the discharge of waves when they reach downstream stations. As a result, it is likely that the arrival time of the wave is also affected.
- ❖ Stage was just as effective as discharge at tracking waves, and it was a better predictor of celerity at a station. Because stage is much easier and cheaper to collect than discharge, it might be possible to maintain gauges even when it is too expensive to maintain rating curves.
- ❖ Although stage is the best descriptor of wave celerity, there are groups of waves that don't follow this relationship, and the relationship breaks down when you consider the movement of individual waves. More work is needed.

Theoretical Background

Most of today's theoretical work on waves in shallow water is based upon a set of equations developed in 1871 by Adhémar Jean Claude Barré de Saint-Venant, after who they were named (eq. 1 and 2).

$$\partial Q/\partial x + \partial A/\partial t - q = 0 \quad \text{(eq. 1)}$$

$$\underbrace{(1/A)\,\partial Q/\partial t}_{\text{local acceleration}} + \underbrace{(1/A)\,\partial(Q^2/A)/\partial x}_{\text{convective acceleration}} + \underbrace{g\partial y/\partial x}_{\text{pressure acceleration}} - \underbrace{g\,(S_0}_{\text{gravity}} - \underbrace{S_f}_{\text{friction}}) = 0 \quad \text{(eq. 2)}$$

For linear features such as streams, the two Saint-Venant equations are used in a one-dimensional form to describe changes in discharge (Q) and cross-sectional area (A) that occur over time (t) along only one dimension of the stream, the length (x). An additional term in the continuity equation (1) includes q, which describes the lateral influx of water entering the system from tributaries or convergent slopes.

The momentum equation (2) can be expressed as five terms. ***Local acceleration*** describes changes in velocity over time, or unsteady flow. The next two terms describe non-uniform flow, or changes in velocity that occur along the length of the channel. These include ***convective accelerations***, caused by constrictions of the cross-sectional area, and ***pressure acceleration***, caused by changes in water depth (y) affected by gravitational acceleration (g). The last two terms describe velocity components that stay constant over time: Accelerations due to the slope of the channel bed (S_0), and frictional losses, as represented by the friction loss slope (S_f).

Together, the two equations describe ***'dynamic'*** waves, and are capable of modelling rivers where the velocity field changes over time due to passing flood waves. The full Saint-Venant equations are used in the HEC-RAS model.

In practice, simplified versions of the Saint-Venant equations are frequently used to describe flood waves. ***'Diffusion'*** wave models drop the initial two inertial terms of the momentum equation, retaining the pressure force, bed slope, and friction slope terms. This simplification can still describe how the peak discharge of a wave might attenuate as it moves downstream, but it only partially describes the ability of the pressure wave to propagate upstream, creating backwater effects.

'Kinematic' waves have the simplest description, only using the last terms from the momentum equation involving bed slope and the energy or friction slope (S_0 & S_f). This description assumes steady, uniform flow, resulting in a wave that moves with constant shape and celerity down a channel. It cannot account for the attenuation of the peak or the dispersion of the main body of the wave and does not account for backwater effects from the wave.

hydrofunctions is a python package for working with hydrology data. You can install it from PyPI like this:

```
$ pip install hydrofunctions
```

Learn more from the manual: **hydrofunctions.readthedocs.io**

or visit on github: **github.com/mroberge/hydrofunctions**